



KINGS



COLLEGE OF ENGINEERING
Punalkulam
DEPARTMENT OF SCIENCE AND HUMANITIES
QUESTION BANK

Subject Code : MA1101

Subject Name: Mathematics-I

Year/Sem: I/I

UNIT-I MATRICES

PART-A

1. Find the constants a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3 and -2 as its eigen values.

2. The product of two eigen values of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16,

Find the third eigen value.

3. Find the sum and product of the eigen values of the matrix

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{pmatrix} \text{ using properties.}$$

4. If the sum of two eigen values and trace of a 3X3 matrix A are equal, find $|A|$.

5. Given that $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$, find the eigen values of A^3 .

6. Find the eigen values of A^{-1} if the matrix A is $A = \begin{pmatrix} 2 & 5 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{pmatrix}$.

7. Two eigen values of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal to 1 each. Find

the eigen values of A^{-1} .

8. If 1 & 2 are the eigenvalues of a 2X2 matrix A, what are the eigenvalues of A^2 and A^{-1} .

9. Let λ be an eigen value of a non-singular matrix A with eigen vector x.

Show that $\frac{1}{\lambda}$ is an eigen value of A^{-1} with eigen vector x.

10. If $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ are the eigen values of an nXn matrix A, then show that $\lambda_1^3, \lambda_2^3, \lambda_3^3, \dots, \lambda_n^3$ are the eigen values of A^3 .

11. Find the sum of the squares of the eigenvalues of $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

12. State Cayley-Hamilton theorem.

13. Give two uses of Cayley-Hamilton theorem.

14. using Cayley-Hamilton theorem find the inverse of $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

15. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 3 & -1 \\ -1 & 5 \end{pmatrix}$.

16. If $A = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$ write A^3 in terms of A and I, using Cayley-Hamilton theorem.

17. Define orthogonal matrix.

18. write down the quadratic form corresponding to the symmetric matrix

$$\begin{pmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{pmatrix}.$$

19. Classify the quadratic forms $x_1^2 + x_3^2$ and $x_1^2 - x_2^2$.

20. Find the index and signature of the quadratic form $x_1^2 + 2x_2^2 - 3x_3^2$.

PART-B

1.a. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ and

hence find the eigen values of $A^2, 5A$ and A^{-1} using properties. **(8)**

b. Find the eigen values and eigen vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$. **(8)**

2.a. Find the eigen values and the eigen vectors of the matrix $\begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$. **(8)**

b. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$. **(8)**

3.a. Find the eigen values and the eigen vectors of the matrix $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$. **(8)**

b. Using Cayley-Hamilton theorem, find A^{-1} given the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$. **(8)**

4.a. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 7 & 3 \\ 2 & 6 \end{pmatrix}$ and hence find A^{-1} and A^3 . **(8)**

b. Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$. **(8)**

5.a. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, find A^{-1} and A^4 using Cayley-Hamilton theorem. **(8)**

b. Verify Cayley-Hamilton theorem and hence find A^{-1} if $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. **(8)**

6.a. Diagonalise $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$. **(8)**

b. Diagonalize the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ using an orthogonal transformation. **(8)**

7.a. Diagonalize $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ by an orthogonal transformation. **(8)**

b. Reduce the matrix $\begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$ to diagonal form. **(8)**

- 8.a. Reduce the quadratic form $x^2 + 2y^2 + z^2 - 2xy + 2yz$ into canonical form. (8)
 b. Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$. (8)
9. Reduce $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction and find the rank, index, signature and the nature of the quadratic form. (16)
10. Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to canonical form through an orthogonal transformation. (16)
11. Reduce the quadratic form to canonical $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ form through an orthogonal transformation. (16)
12. Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ to canonical form through an orthogonal transformation and hence show that it is positive semi-definite. (16)

UNIT - II

THREE DIMENSIONAL GEOMETRY

PART - A

1. Find the angle between the lines $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$ and $\frac{x-4}{2} = \frac{y-1}{1} = \frac{z+6}{2}$
2. Find the angle between the lines whose direction cosines are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ & $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.
3. Find the direction cosines of the line joining the points (2,3,-6) and (3,-4,5).
4. Find the equation of the line joining the points (1,2,3) and (-3,4,5).
5. Find the acute angle between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$.
6. Prove by direction ratios, the points (1,2,3), (4,0,4), (-2,4,2) are collinear.
7. Find the direction cosines of a line perpendicular to the two lines whose direction ratios are (1,2,3) and (-2,1,4).
8. Find the projection of the segment joining A(1,2,3) and B(6,7,9) on the line whose direction ratios are (1,2,-3).
9. Find the values of K, if the lines $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{K}$ and $\frac{x-3}{K} = \frac{y-2}{3} = \frac{z-4}{5}$ are coplanar.
10. Find the centre and radius of the sphere $2(x^2 + y^2 + z^2) + 6x - 6y + 8z + 9 = 0$.
11. Find the equation of the sphere concentric with $x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$ and passing through the point (-2,1,-5).
12. Find the equation of the sphere whose centre is same as that of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 7 = 0$ and which passes through the point (1,-1,1).

13. Write down the equation of the sphere whose diameter is the line joining the points (1,1,1) and (-1,-1,-1).
14. Find the centre and the radius of the sphere
 $7(x^2 + y^2 + z^2) + 28x - 42y + 56z + 3 = 0$.
15. Check whether the two spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$
and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ intersect each other orthogonally.
16. Find the equation of the sphere having the circle
 $x^2 + y^2 + z^2 = 9$ and $x - 2y + 2z = 5$ as a great circle.
17. Find the equation of the tangent plane at the point (1,-1,2)
to the sphere $x^2 + y^2 + z^2 - 2x + 4y + 6z - 12 = 0$.
18. Write down the general equation of the cone whose vertex is at the origin.
19. Find the equation of the cone with vertex at the origin and which passes
through the curve $x^2 + y^2 = 4, z = 2$.
20. Write down the equation of the right-circular cylinder whose axis is the z-axis
and radius "a" units.

PART-B

1. a) Find the length and the equation of shortest distance between
The lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. (8)
- b) Show that the lines $\frac{x-4}{2} = \frac{y-5}{3} = \frac{z-6}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$
are coplanar. (8)
2. a) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$
are coplanar and find their point of contact. (8)
- b) Prove that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = z+2$ and $x = \frac{y-7}{-3} = \frac{z+7}{2}$
intersect. Find the co-ordinates of the point of intersection and equation of the
plane containing them. (8)
3. a) Find the angle between the straight lines whose direction cosines are given
by the relation $3l+m+5n=0$ and
 $6mn-2nl+5lm=0$. (8)
- b) Find the equation of the sphere described on the line joining
the points (2,-1,4) and (-2,2,-2) as diameter. Find also the area of the circle in
which the sphere is cut by the plane $2x+2y-z=3$. (8)
4. a) Find the equation of the sphere passing through the points (1,1,-2) and (-
1,1,2) and having its centre on the line
 $x+y-z-1=0=2x-y+z-2$. (8)
- b) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$
 $2x + 3y + 4z = 5$ and the point (1,2,3). (8)
5. a) Show that the plane $2x-2y+z=9$ touches the sphere
 $x^2 + y^2 + z^2 + 2x + 2y - 7 = 0$ and find the point of contact. (8)
- b) Find the equation of the tangent plane to the sphere

- $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$ at the point (1,2,3). Also find the equation of the normal to the sphere at (1,2,3). (8)
6. a) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0$ which passes through the line $9x - 3y + 25z = 0 = 3x + 4z + 9$. (8)
- b) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point (1,-2,1) and also cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$. (8)
7. a) Prove that a point at which the sum of the squares of whose distances from the planes $x + y + z = 0$, $x - z = 0$, $x - 2y + z = 0$ is 9, lies on the sphere $x^2 + y^2 + z^2 = 9$. (8)
- b) Find the area of the circle in which the sphere $x^2 + y^2 + z^2 + 12x - 2y - 6z + 30 = 0$ is cut by the plane $x - y + 2z + 5 = 0$. (8)
8. a) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ and $x + 2y + 2z + 7 = 0$. (8)
- b) Find the equation of the cone with vertex at the origin and the guiding curve is the circle $x^2 + y^2 + z^2 + 4x + 2y - 6z + 5 = 0$, $2x + y + 2z + 5 = 0$. (8)
9. a) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. (8)
- b) Show that the equation to the right-circular cone whose vertex is O, axis OZ and semivertical is α is $x^2 + y^2 = z^2 \tan^2 \alpha$. (8)
10. a) Find the equation of the cylinder whose generator are parallel to the line $x = \frac{-y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. (8)
- b) Find the equation of the right-circular cylinder of radius 2 whose axis passes through the point (1,2,3) and has direction cosines proportional to 2,-3,6. (8)

UNIT III DIFFERENTIAL CALCULUS PART - A

1. Find the radius of curvature of the curve $y = \log \sin x$ at $x = \pi/2$.
2. Find the radius of curvature of the curve $y = e^x$ at the point where it crosses the y-axis.
3. Find the radius of curvature of the curve $xy = c^2$ at (c,c).
4. Find the curvature at (3,-4) to the curve $x^2 + y^2 = 25$.
5. What is the curvature of $x^2 + y^2 - 4x - 6y + 10 = 0$ at any point on it

6. Define the curvature of a plane curve and what is the curvature of the straight line.
7. Find the radius of curvature at any point on the curve $r = e^\theta$
8. Find the radius of curvature at $y = 2a$ on the curve $y^2 = 4ax$.
9. Find the radius of curvature of the curve $y = a \cosh(x/a)$ at the point where it crosses the y axis.
10. Find the radius of curvature of the curve $y = \clog(\sec(x/c))$.
11. Find the radius of curvature at $x = \pi/2$ on the curve $y = 4\sin x - \sin 2x$.
12. Write any two properties of evolute.
13. Define evolute.
14. Find the envelope of the lines $x/t + yt = 2c$, t being the parameter.
15. Show that the family of straight lines $2y - 4x + \lambda = 0$ has no envelope where λ is the parameter.
16. Find the envelope of $\frac{x}{a} \cos\theta + \frac{y}{b} \sin\theta = 1$, where θ is the parameter.
17. Find the envelope of the family of straight lines $x \cos\theta + y \sin\theta = 6$, where θ is the parameter.
18. Find the envelope of the family of straight lines $x \cos\alpha + y \sin\alpha = a \sec\alpha$, where α is the parameter.
19. Find the envelope of the family $1 - x^2 + (y-k)^2 = 0$ where k is the parameter.
20. Find the envelope of $x^2 + y^2 - ax \cos\theta - by \sin\theta = 0$ where θ is a parameter.

PART – B

1. (a) Find the radius of curvature of the curve $x = 3a \cos \theta - a \cos 3\theta$,
 $y = 3a \sin \theta - a \sin 3\theta$ at θ . (8)
- (b) Find the radius of curvature of the parabola $x = at^2$, $y = 2at$ at t . (8)
2. (a) For the curve $x = a(\cos \theta + \theta \sin\theta)$, $y = a(\sin\theta - \theta \cos\theta)$, prove that the radius of curvature is $a\theta$. (8)
- (b) Find the radius of curvature at any point $(a \cos 3\theta, a \sin 3\theta)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$. (8)
3. (a) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a \cos \theta/2$. (8)
- (b) Find the center of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $(a/4, a/4)$. (8)
4. (a) Find the circle of curvature of the parabola $y^2 = 12x$ at the point $(3, 6)$. (8)

- (b) Show that the circle of curvature of $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $(a/4, a/4)$ is
 $(x-3a/4)^2 + (y-3a/4)^2 = a^2/2$. (8)
5. (a) Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (8)
- (b) Show that the equation of the evolute of the parabola $x^2 = 4ay$ is
 $4(y-2a)^3 = 27ax^2$. (8)
6. (a) Find the equation of the evolute of the parabola $y^2 = 4ax$. (8)
- (b) Find the equation of the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)
7. (a) Obtain the equation of the evolute of the curve $x = a(\cos \theta + \theta \sin \theta)$,
 $y = a(\sin \theta - \theta \cos \theta)$. (8)
- (b) Find the evolute of the four cused hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$. (8)
8. (a) Show that the evolute of the cycloid $x=a(\theta - \sin \theta), y=a(1-\cos \theta)$ is another cycloid. (8)
- (b) Find the evolute of the rectangular hyperbola $xy = c^2$. (8)
9. (a) Prove that the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by $a + b = c$ is $\sqrt{x} + \sqrt{y} = \sqrt{c}$. (8)
- (b) Find the equation of the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by the relation $a^2 + b^2 = c^2$ and c is a constant. (8)
10. (a) Find the envelope of $y \cos \theta - x \sin \theta = a \cos 2 \theta$ where θ is a parameter. (8)
- (b) Find the envelope of $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$, θ is a parameter. (8)

UNIT- IV
FUNCTIONS OF SEVERAL VARIABLES
PART-A

1. If $x = u(1-v), y = uv$ find $\frac{\partial(u, v)}{\partial(x, y)}$.
2. Find the minimum point of $f(x,y) = x^2 + y^2 + 6x + 12$.
3. Find the stationary point of $f(x,y) = xy + \frac{9}{x} + \frac{3}{y}$.

4. Find Taylor's series expansion of $e^x \sin y$ near the point $(-1, \frac{\pi}{4})$ up to the first degree terms.
5. If $u=f(x-y, y-z, z-x)$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
6. Find $\frac{du}{dt}$, if $u=x^3y^2+x^2y^3$, where $x=at^2, y=2at$, using partial derivatives.
7. Find the stationary points of the function $f(x,y)=x^3+y^3-12xy$.
8. If $u = a \cosh x \cos y$, $v = a \sinh x \sin y$, then show that $\frac{\partial(u, v)}{\partial(x, y)} = \frac{1}{2} a^2 (\cosh 2x - \cos 2y)$,
9. If $u=x+y$ and $y=uv$ find the jacobian $\frac{\partial(x, y)}{\partial(u, v)}$.
10. Find $\frac{du}{dt}$, if $u = \frac{x}{y}$, where $x=e^t$, $y = \log t$.
11. Find $\frac{\partial(u, v)}{\partial(r, \theta)}$, if $u=2xy$, $v = x^2-y^2$, $x= r \cos \theta$, $y=r \sin \theta$.
12. If $z = \sin^{-1}(x-y)$, $x = 3t, y=4t^2$ find $\frac{dz}{dt}$.
13. If $u= f(x+ay)+g(x-ay)$ prove that $\frac{\partial^2 u}{\partial y^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.
14. Find the Taylor's series expansion of x^y near the point $(1,1)$ up to the second degree terms.
15. If $u=e^x y z^2$ find du .
16. If $x = r \cos \theta$, $y = r \sin \theta$ find $\frac{\partial(r, \theta)}{\partial(x, y)}$.
17. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$.
18. If $x = u(1+v)$ and $y=v(1+u)$, find $\frac{\partial(x, y)}{\partial(u, v)}$.
19. If $u = \frac{x}{y}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.
20. Find the stationary points of x^2-xy+x^2-2x+y .

PART – B

- 1.(a) Find the maxima and minima for xy^2z^3 subject to $x+y+z=6$. (8)
- (b) Are the functions $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}(x) + \tan^{-1}(y)$ functionally dependent ?
If so, find the relation between them. (8)

- 2.(a) Expand $f(x,y) = 4x^2+xy+6y^2+x-20y+21$ in Taylor's series about $(-1,1)$. (8)
 (b) If $u = 4x^2+6xy$, $v = 2y^2+xy$, $x=r\cos\theta$, $y=r\sin\theta$. Evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$. (8)
- 3.(a) Find the minimum value of xy^2z^2 subject to $x+y+z=24$ using Lagrange multiplier. (8)
 (b) Expand $e^x \log(1+y)$ in powers of x and y up to the terms of third degree. (8)
- 4.(a) Given the transformation $u=e^x \cos y$, $v = e^x \sin y$ and that ϕ is a function of u and v also of x and y , prove that

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (u^2 + v^2) \left(\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi}{\partial v^2} \right)$$
 (8)
- (b) Investigate the maxima and minima, if any, of the function $y^2+4xy+3x^2+x^3$ (8)
- 5 (a) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ prove $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. (8)
 (b). Using Lagrange's multiplier method, determine the maximum capacity of a rectangular tank, open at the top, if the surface area is 108 sq.m. (8)
- 6(a). If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. (8)
 (b) Expand $x^2y + \sin y + e^x$ in powers of $(x-1)$ and $(y-\pi)$ through quadratic forms. (8)
- 7(a). Find the maximum and minimum values of $2(x^2-y^2)-x^4+y^4$. (8)
 (b) Examine the function $f(x,y) = x^3y^2(12-x-y)$ for extreme values. (8)
- 8 (a). Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if
 $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_3x_1}{x_2}, y_3 = \frac{x_1x_2}{x_3}$. (8)
 (b). If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \cot u$. (8)
- 9(a). Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's expansion. (8)
 (b). Examine $f(x,y) = x^3y^3 - 12x - 3y + 20$ for its extreme values. (8)
- 10(a). Using Taylor's expansion, express $f(x,y) = e^{ax} \cos(by)$ in powers of x and y up to second degree terms. (8)
 (b) Obtain terms up to the third degree in the Taylor series expansion of $e^x \sin y$ around the point $[1, \pi/2]$ (8)

UNIT-V

ORDINARY DIFFERENTIAL EQUATIONS

PART-A

1. Solve $(D^2 + 4)y = 0$.
2. Find the P.I of $(D^2-2D+5)y = e^x \sin 2x$.
3. Solve $\frac{dx}{dt} - y = 0; \frac{dy}{dt} + x = 0$
4. Write Euler's homogeneous linear differential equation. How will you convert it to a linear differential equation with constant coefficients?
5. Solve $(D^2+2D+1)y=e^{-x}$.

6. Find the P.I of $(D^3+8)y = \cosh 2x$.
7. Solve $(x^2D^2-xD+1)y=0$.
8. Solve $\frac{d^2y}{dx^2} = y$
9. If $x = e^z$, express $\frac{d^2y}{dx^2}$ in terms of the derivatives of y w.r.to z .
10. Solve $(D^2+D+1)y=0$
11. Find the P.I. Of the $(D^2 + 1)y = x^3$.
12. Find the P.I. of $(D^2 + 4)y = \cos 2x$.
13. Solve $(x^2D^2+4xD+2)y=0$.
14. Find the P.I. Of the $(D^3 - 1)y = e^{2x}$.
15. Find the P.I. Of the $(D - 1)^2y = e^x \sin x$.
16. Find the P.I. Of the $\frac{d^2y}{dx^2} = xe^x$.
17. Solve $(D^3 - 3D^2 + 3D - 1)y = x^2e^x$.
18. Solve $(D^3 - 6D^2 + 11D - 6)y = 0$
19. Find the C.F. of $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$.
20. Solve $((3x+2)^2D^2 - (3x+2)D + 1)y = 0$.

PART - B

1. a) solve $(D^2+2D-1)y=x^2+e^{2x}$ (8).
 b) Solve $(x^2D^2-2xD-4)y=32(\log x)^2$. (8)
2. a) Solve $(D+4)x+3y = t$
 $(D+5)y+2x = e^{2t}$ (8)
 b) Solve $y'' + y = \sec x$ by method of variation of parameters (8).
3. a) Solve by method of variation parameters $y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1$ (8)
 b) Solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ by using method of variation of parameters.
4. a) Solve $(D^2+6D+8)y = e^{-2x} + \cos^2 x$ (8)
 b) Solve $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = \sin(\log x)$ (8).
5. a) Solve $\frac{dx}{dt} + y = e^{2t}$, $\frac{dy}{dt} + 4x = t$, given that $x(0) = 2$ and $y(0) = \frac{1}{4}$ (8)
 b) Solve $(x^2D^2+4XD+2)y=x \log x$ (8).
6. a) Solve $(D^2+5D+4)y=e^{-x}\sin 2x+x^2+1$. (8)

b) Solve $\frac{d^2y}{dx^2} + y = x \cos x$, by using method of variation of parameters (8).

7 a). Solve $\frac{dx}{dt} + 2y = -\sin t, \frac{dy}{dt} - 2x = \cos t$. (8)

b) Solve $(2x+3)^2 y'' - (2x+3)y' - 12y = 6x$. (8)

8. a) Solve $(D^2 - 2D + 2)Y = e^x x^2 + 5 + e^{-2x}$. (8)

b) Solve $(D^2 + 4D + 3)y = e^{-x} \sin x + xe^{3x}$. (8)

9. a) Solve by method of variation of parameters $\frac{d^2y}{dx^2} + y = x \sin x$ (8).

b) Solve $(x+2)^2 \frac{d^2y}{dx^2} - (x+2) \frac{dy}{dx} + y = x+2$. (8)

10. a) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = x^2 + \cos(\log x)$ (8).

b) Solve $(D^2 - 2D + 1)Y = xe^x \sin x$ (8).